HW #2: Discrete Structures

CS 250: Discrete Structures
Fall 2015

[100pts] DUE 11am 10/20/2015 (Tuesday)

Instructions

- This HW covers Lectures 2.1 to 2.5, which are drawn from Chapter 2 of Rosen. Please review these thoroughly before starting to work on the HW.
- All soft copy submissions must be turned in on Blackboard Learn. Name your files as “Lastname_Firstname_HW2”. Please make sure that you have correctly submitted/uploaded the files.
- You can submit hand-written hard copies, if you wish. However, we would prefer that you submit soft copies. If you choose to submit hard copies in any case, please turn them into my mailbox in the CS office. Please also make sure that your hand-written solutions and handwriting is legible and easy to understand.
- You must submit by the deadline – 11 am on 10/20/2015 (Tuesday). This applies to both soft and hard copy submissions. Late submissions will not be graded.
- Please submit early to avoid any last minute issues.
- Please do start working on the homework early and do not wait until the deadline.
- You will be graded based on the correctness of your answer and also on the steps that you took to come to that answer. Please show all your work, whenever asked.
Problem 1 [20pts]

(1) [6pts] List the elements of the following sets:
   1. [2pts] \{x: x is a real number such that \(x^2 = 9\)\}
   2. [2pts] \{x: x is a positive integer less than 15\}
   3. [2pts] \{x: x is an integer such that \(x^2 = 78\)\}

(2) [6pts] A, B, C and D are four sets such that \( A \subseteq C \) and \( B \subseteq D \). Show that \( A \times B \subseteq C \times D \).

(3) [4pts] Find the power set of each of the following sets, where a and b are distinct elements:
   1. [2pts] \{a, b\}
   2. [2pts] \{\emptyset, \{a\}\}

(4) [4pts] Let \( A = \{a, b, c, d\} \), and \( B = \{y, z\} \). Find the following Cartesian products:
   1. [2pts] \( A \times B \)
   2. [2pts] \( B \times A \)

Problem 2 [20pts]

(1) [8pts] Let \( A = \{1, 2, 3, 4, 5\} \) and \( B = \{0, 3, 6\} \). Find the following:
   1. [2pts] \( A \cup B \)
   2. [2pts] \( A \cap B \)
   3. [2pts] \( A \setminus B \)
   4. [2pts] \( A \oplus B \)

(2) [12pts] Prove that \( A \cup B = A \cap B \) using the following approaches (show all of your work):
   1. [6pts] by showing each side is a subset of the other side
   2. [6pts] by using a membership table

Problem 3 [30pts]

(1) [6pts] Argue whether or not the following \( f(x) \) defined from \( \mathbb{R} \) (set of real numbers) to \( \mathbb{R} \) is a function:
   1. [3pts] \( f(x) = 1/x \)
   2. [3pts] \( f(x) = \sqrt{x} \)

(2) [12pts] Argue whether or not the following functions, from \( \mathbb{R} \) to \( \mathbb{R} \), are bijections
   1. [3pts] \( f(x) = 2x + 1 \)
2. [3pts] \( f(x) = x^2 + 1 \)
3. [3pts] \( f(x) = x^3 \)
4. [3pts] \( f(x) = (x^2 + 1)/(x^2 + 2) \)

(3) [3pts] What is \( f(S) \) where \( f(x) = \left\lfloor x^2 / 3 \right\rfloor \) and \( S = \{-2, -1, 0, 1, 2, 3\} \)?

(4) [5pts] Find \( f \circ g \) and \( g \circ f \), where \( f(x) = x^2 + 1 \) and \( g(x) = x + 2 \) are functions defined from \( \mathbb{R} \) to \( \mathbb{R} \). Show all your work.

(5) [4pts] Find the inverse function of \( f(x) = x^3 + 1 \), defined from \( \mathbb{R} \) to \( \mathbb{R} \). Show all your work.

Problem 4 [30pts]

1) [10pts] Assume that the population of the world in 2010 was 6.9 billion and is growing at the rate of 1.1% a year. Find an explicit formula for the population of the world \( n \) years after 2010. Also, what will be the population of the world in 2030? Show all your work.

2) [10pts] Find a formula to express the elements of the following sequences. Show all your work.
   1. [5pts] 3, 6, 12, 24, 48, 96, 192,…
   2. [5pts] 1, 3, 15, 105, 945, 10395,…

3) [10pts] Compute the following sums. Show all of your work.
   1. [5pts] \( \sum_{k=1}^{5} (2k + 1) \)
   2. [5pts] \( \sum_{j=0}^{8} (2^{j+1} - 2^j) \)